Hafod y Rhedrwydd micro-hydro scheme.

Weir design calculations v.3a.

Contents

Design aims:

(1) provide attached flow over the Coanda screens so that they can extract water as intended

(2) provide hardware safeguards to protect the stream flow:

- no more than 70% "zone 1" extraction (in fact the chosen design is limited to 60% due to other design constraints)
- no more than 0.6 litre/sec extraction beyond Q95 (stream flow 4 L/s or less)
- no more than a nominal 0.05 L/s extraction should the stream flow ever fall below 0.9 L/s (most unlikely with predicted Q99.9 = 1.8 L/s, really an emergency provision).

(3) improve access for fish so that they can pass up and downstream without hindrance or injury.

Matlab code is provided for all the figures in this submission. The "m-file" names after each graph indicate the code that generates it. The code is annotated with explanatory comments.

Location mapping.

A Leica Runner 24 automatic level was used to survey the stream up to 20 m each side of the abstraction point (Figure 1). Measurements of bed height, and bank position were taken together with water depth and edge location at 22 measurement stations. The stream flow was estimated (by eye) to be about 7 litres/second on the survey day.

The relevant Ordnance Survey MasterMap and satellite imagery were purchased from Blackwells Map Shop (Leeds), imported into DraftSight and superimposed. Recorded coordinates were saved in Matlab as a DXF file and imported into DraftSight; the survey positions were then aligned with the satellite image and bank outlines were added using the measure widths (drawing HyR_181006A).

Figure 1. Surveying the extraction point.

3D modelling and general arrangement.

The bank outlines were exported from DraftSight as another DXF file. This was loaded into AutoDesk Fusion 360 as the basis for a 3D CAD model (necessary to investigate possible plunge pool positions and illustrate the general appearance of the abstraction system).

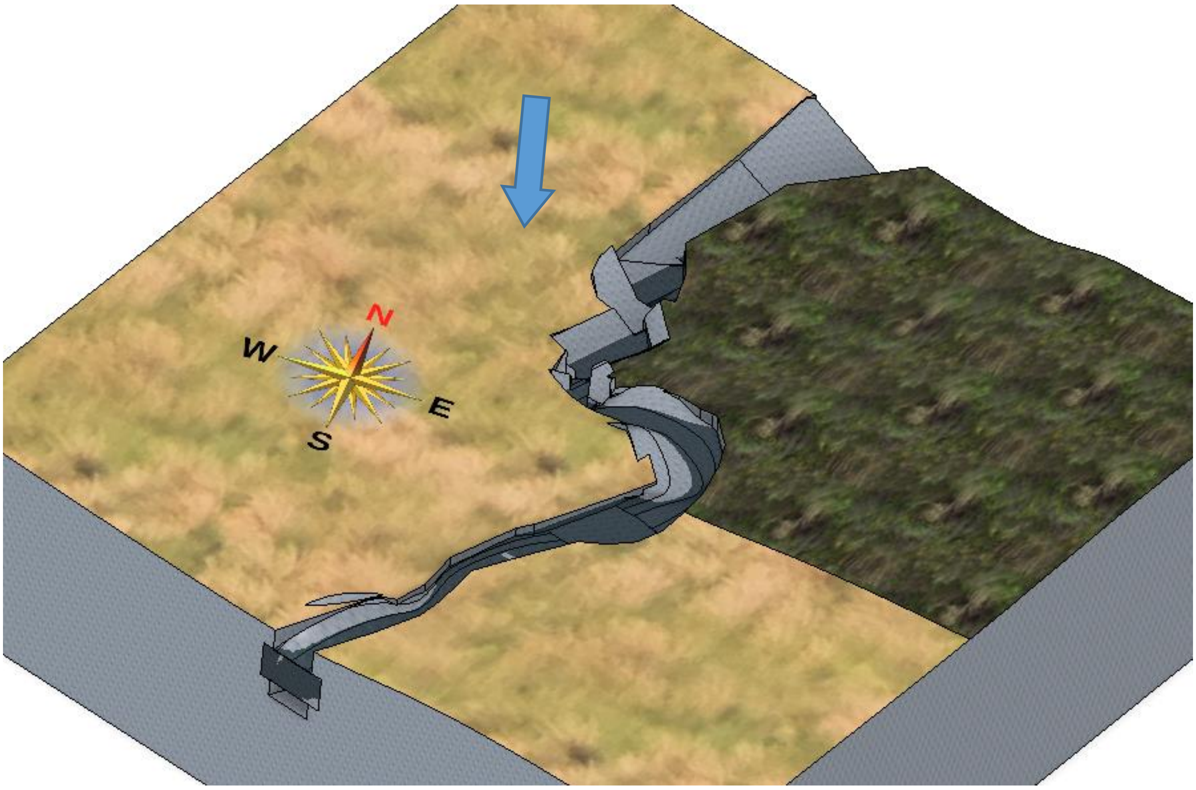


Figure 2. 3D model of the abstraction point surroundings. Surface vegetation (heather or grass) is indicated by texture maps. The rock strata slope down at 28° to the horizontal. Arrow shows Figure 3 view direction.

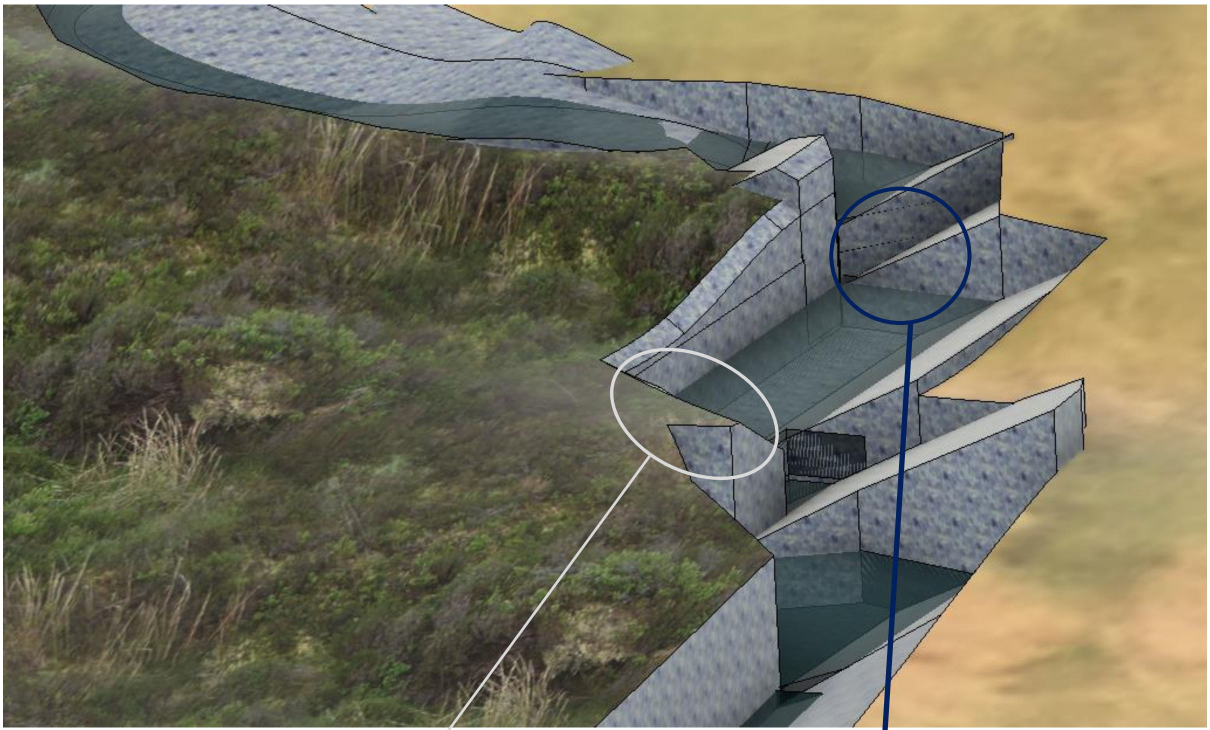


Figure 3. Abstraction point showing sloping strata and existing rock pools. Drawing HyR_181006B. The hatched area to right of center is just a drawing artifact.

Figure 4. Photograph of extraction point. The Coanda screens will be mounted against the face of the existing waterfall (circled). The height of the grassy bank on the left-hand side is only loosely represented in the model (grey circle). To give an indication of scale, the length of the rock wall is approximately 4 m.

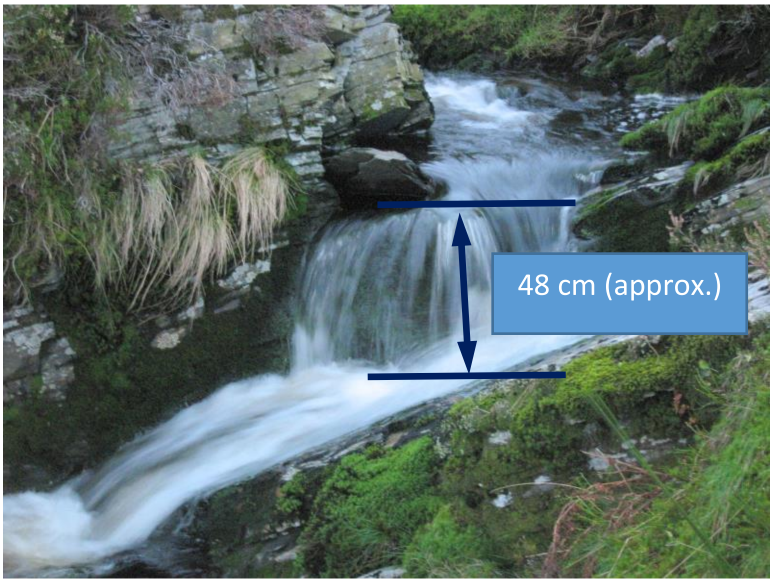


Figure 5. Close-up of waterfall. The drop of 48 cm onto a sloping rock face might make this an obstacle to fish: it is hoped that the proposed extraction system will allow easier passage.

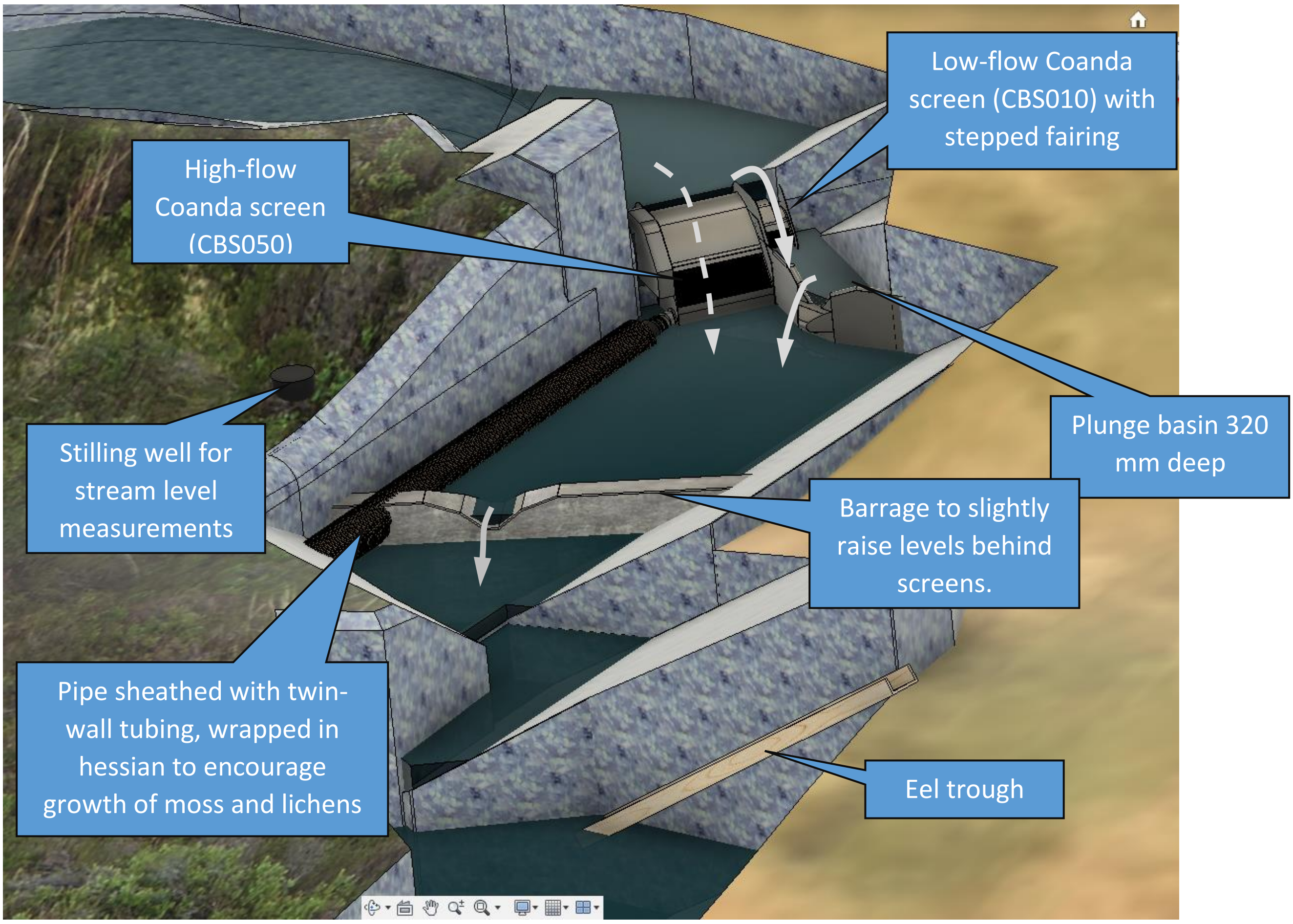


Figure 6. Abstraction system. Fish can progress up and downstream in three stages (black arrows), with jumps of less than 25 cm. Water levels shown for stream flow of 4 litres/sec, Q95. At higher stream levels, water starts to flow over the high flow screen (dashed arrow). Drawing HyR_181006B.

Flow calculations for weirs.

All the weirs are broad-crested, avoiding (for instance) sharp-edged V-notch weirs which might be hazardous to fish. The plunge basin has stainless-steel walls (painted to match the surroundings) on two sides but these have a 40 mm diameter tube welded to the upstream face to provide a smooth edge, so it is effectively broad-crested.

The flow rate is calculated as $Q = C_d L \sqrt{g} \left(\frac{2}{3} H \right)^{\frac{3}{2}}$ (*Water Measurement Manual* eqn. 2-39).

H is the upstream total head relative to the weir crest. The discharge coefficient ($C_d \approx 1$) depends slightly on the upstream slope angle and flow curvature. No attempt has been made to accurately predict C_d because the possible variations are not large enough to significantly change the predicted water depths; instead, model tests and in-situ adjustments to fairing height will be used to ensure the design performance is achieved in practice.

If the crest slopes such that the total head varies from H_1 to H_2, integration of the flow equation gives a total flow $Q = \frac{2}{5} C_d L \sqrt{g} \left(\frac{2}{3} H \right)^{\frac{3}{2}} \left(\frac{1 - \left(\frac{H_2}{H_1} \right)^{\frac{5}{2}}}{1 - \left(\frac{H_2}{H_1} \right)} \right)$

e.g. for a triangular section, the total flow is $2/5$ of the equivalent rectangular slot flow rather than the $1/2$ that might be expected based purely on the area change. The code uses this equation to model the sloping rock slabs beside the screens and the trapezoidal notches in the plunge basin and barrage.

The Coanda screens use side plates to suppress sideways contraction of the flow. Contraction effects have not been modelled in the simulation because the head is generally small compared to crest length under the low flow conditions used to determine heights.

Crest heights above Coanda screens

Extraction will use a pair of Coanda screens, Elgin models CBS0.10 and CBS0.50:

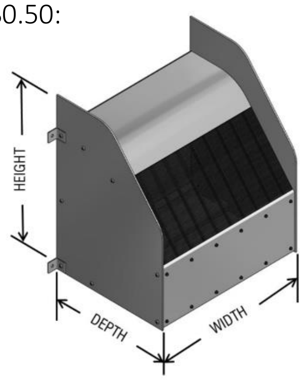


Table 1. Elgin screen dimensions:

Elgin model:	Max Litres/sec	Depth	Width	Height
CBS0.10-1	2.83	8”	12”	11”
CBS0.50-1	14.2	15”	24”	22.5”

The screens have a tight radius leading to the sloping screen element. At high flow rates this is likely to cause separation of the water, with the nappe springing clear of the screen. To avoid this, ogee-profiled fairings will be attached above each screen box.

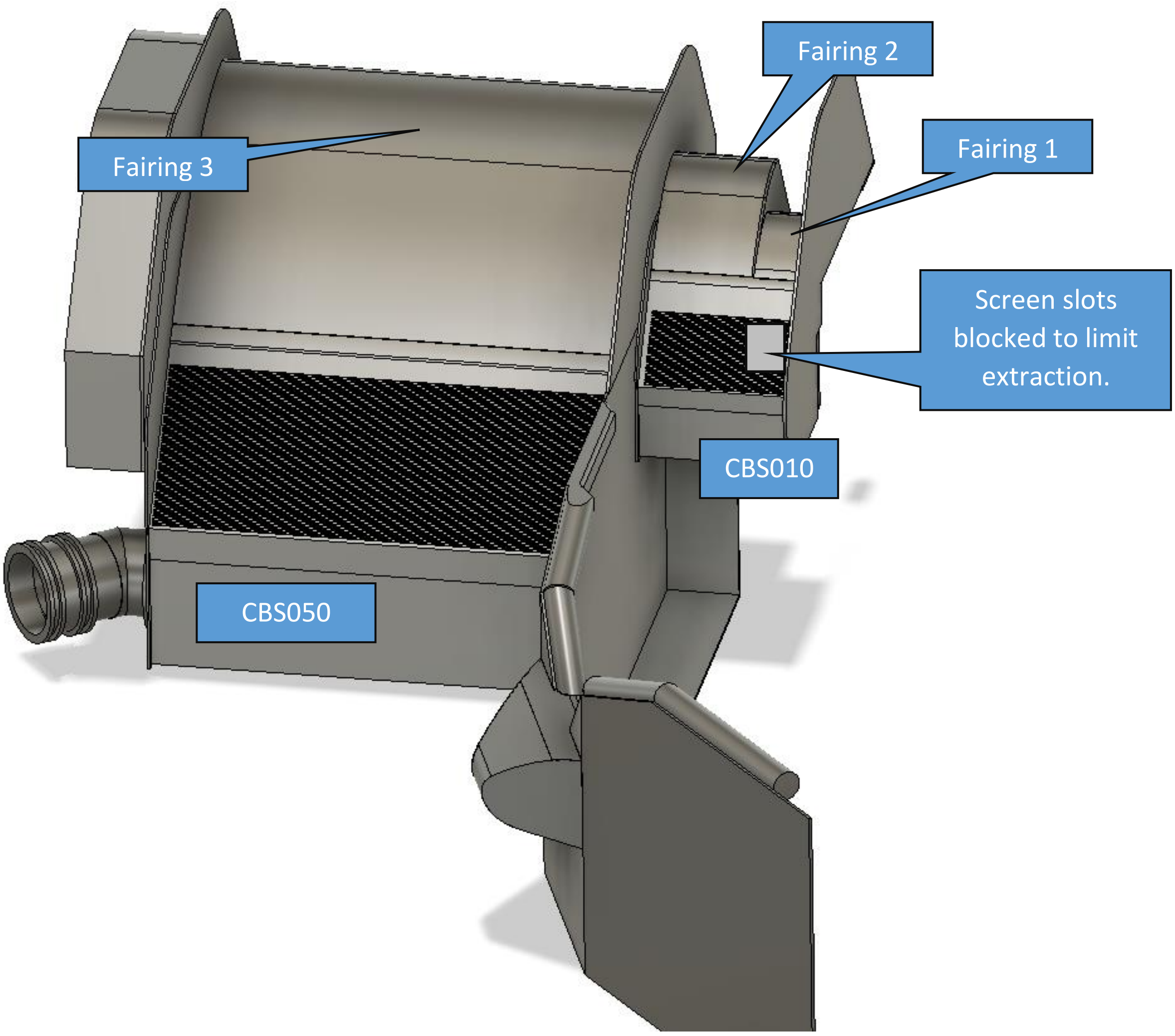


Figure 7. Arrangement of screen boxes and fairings. Drawing HyR_181006C shows further details including widths and relative crest heights. The screen under fairing 1 has only 5 of its 33 slots open, such that the maximum extractable flow is 0.05 litre/sec; the others will be sealed using PermaBond MT382 epoxy. The invert level is approximately 40 cm below the crest of fairing 3.

The fairings will sit on spacers so the height can be varied slightly if required to adjust the transition from one flow regime to another. Crest heights and ogee profiles have been designed using a Matlab simulation (Figures 8 and 9, *run_weir_analysis.m*).

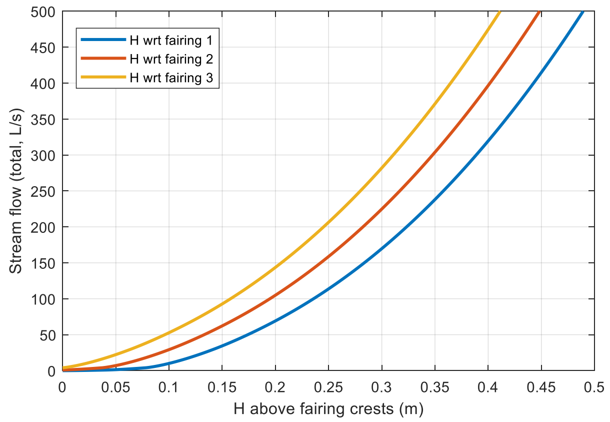


Figure 8. Stream flow as a function of upstream head. (For convenience, flow rate is calculated in terms of head, not vice-versa).

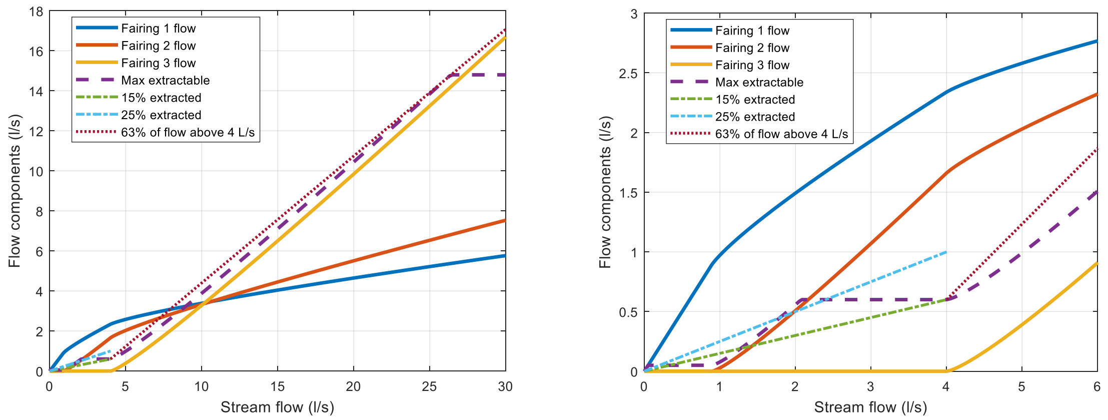


Figure 9. (a) Flow components as a function of stream flow showing split over each fairing and the maximum extractable through the screens. The CBS050 screen (fairing 3) has a maximum capacity of 14.2 litres/sec. (b) Enlarged view showing extraction regulation at low stream flows.

The crest heights are defined in *weir_defs1.m* as follows:

```
ip = 1;  % allows one to test multiple variants
weir(ip).a = NaN * [1 1 1 1 1]; % no upper sluice plate
weir(ip).b = [0.063 0.137 0.6 0.05 1.35]; % fairing widths:
%    fairing 1, fairing 2, fairing 3
%    left buttress, right buttress
weir(ip).dC = [0 0.041 0.078 0.14 0.14]; % crest height increments
weir(ip).thetad = [0 0 0 0 18.2]; % slope of crest
weir(ip).Cdf = [1 1 1 1 1]; % Cd for free-surface flow
weir(ip).Cdo = [1 1 1 1 1]; % Cd for orifice flow
weir(ip).H = (0:0.0004:0.5)'; % vector of total head
weir(ip).HW_limits = [0.05E-3 Inf     14.2E-3   0 0
                  Inf     0.6E-3  Inf       Inf Inf];
% extraction limits through screens of 0.05 and 14.2 L/s under fairings
% 1&2, cumulative limit via orifice for fairings 1+2
weir(ip).SW_limits = []; % these rates achieved without relying
% on software control of the spear valve!
(etc)
for i = 1:length(weir)
```

```
    weir(i) = weirflows(weir(i));
    %function weirflows does the calculation, adds output fields
end
```

At stream flows below 4 l/s no flow passes over fairing 3. The water collected in the smaller (CBS010) screen box has to pass through an orifice (Figure 10) to reach the larger screen box and thence the penstock. This orifice is designed to limit the flow to 0.6 l/s. See *Justification for the extraction regime.pdf* for explanation of the 0.6 L/s choice.

The standard CBS010 screen is 12" wide, compared to the 24" wide CBS050. 12" is unnecessarily wide and would lead to the CBS050 not passing more than $2/3$ of the stream flow, even if there were no difference in crest levels. The CBS010 box will be reduced in width to 200 mm to give a better match to the extraction requirements.

Stream flows below 1.8 L/s (Q99.9) are very unlikely to occur. Just in case the predictions are wrong, however, the blocked slots under fairing 1 start coming into effect at stream flows below 2.1 litres/sec to limit the maximum extraction rate (Figure 9(b)). At stream flows below 0.9 litre/sec, no flow at all passes over fairing 2 and the extraction is limited to a nominal 0.05 litre/sec (to satisfy domestic requirements only).

The maximum extractable flow is 14.8 litres/sec (14.2 l/s from the CBS050-1 screen plus 0.6 l/s through the orifice from the CBS010 screen). This value has been used in form WRB. At the lowest stream flow for which 14.8 l/s could be extracted, the maximum possible extracted flow is

$$0.6+0.63\left(Q_{stream}-4\right) \text{ litres/sec}$$

i.e. taking 63% of the flow above Q95 (dotted line in Figure 9(a)).

It seems unlikely however that flows in excess of 12 litres/sec (62% above Q95) will actually be used since the penstock pressure drop at higher flows leads causes a reduction in output voltage that would require a more complex control system.

More sophisticated regulation of the abstraction regime would be possible, if necessary, using software to monitor the flow passing over the barrage and to adjust the spear valve accordingly.

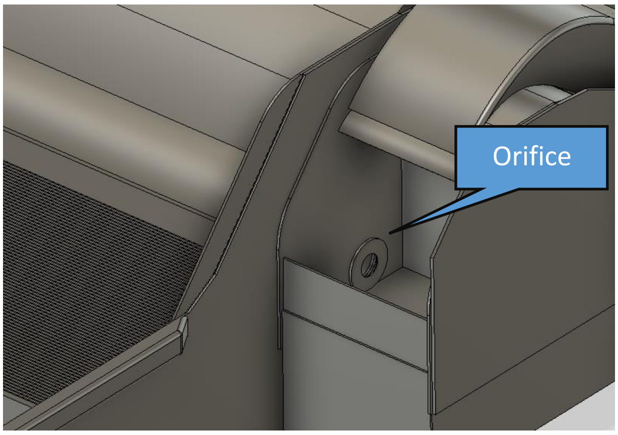


Figure 10. CBS010 box with screen removed to show flow-limiting orifice (27 mm diameter, chamfered at rear to function as an orifice plate). Calculation assumes the box will fill to a height of 149 mm above the orifice centre-line; C_d = 0.61.

Prior to installation the Warwick University tilting flume (Fig 11) will be used to test a scale model of fairings 1&2 to confirm crest levels, since contraction effects and non-uniform approach flow inevitably introduce a small degree of uncertainty.

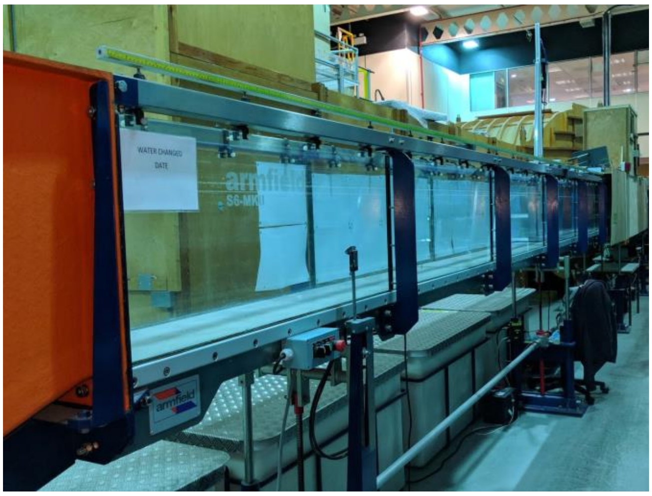


Figure 11. Tilting flume (300 mm wide, flows up to 32 litres/sec).

Post-installation the construction bypass syphon will be used to set a 4 litre/sec stream flow so that the flow split between screens can be confirmed and fairings adjusted slightly in height or width if required.

Calculation of ogee fairing profile above Coanda screens

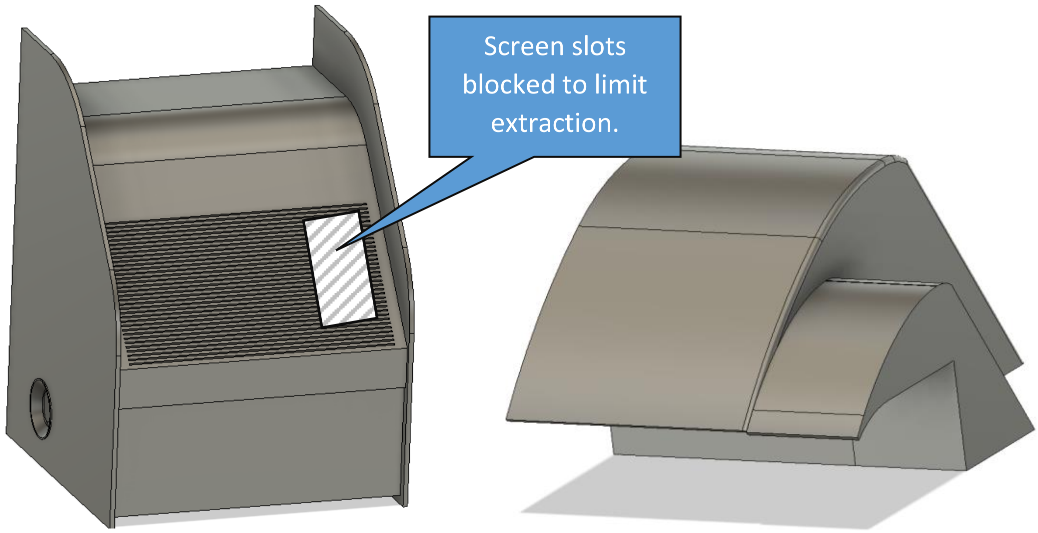


Figure 12. Fusion 360 models of the CBS010 screen box and its fairings. The extreme low flow protection was originally planned as a separate 45 mm wide slot: this has now been incorporated into the screen fairing to provide a more fish-friendly configuration.

Ogee spillways for dams are designed by imagining the shape of a free-flying nappe of water which, accelerated by a head of water exceeding anything likely to be experienced in practice, separates from the crest of a weir and describes a parabolic arc downwards (*Design of Small Dams*, 1987).

Provided the spillway or fairing curve lies above profile of the underside of the nappe, the water should stay in contact with the surface. If the surface is lower than the imaginary nappe, separation is likely to occur or, if not, sub-atmospheric pressures leading to lifting forces and possible cavitation.

The Small Dams formula for nappe profile is $\dfrac{y}{H} = -K\left(\dfrac{x}{H}\right)^n$ where coefficients K and n are provided in graphical form in terms of the upstream velocity head h_a:

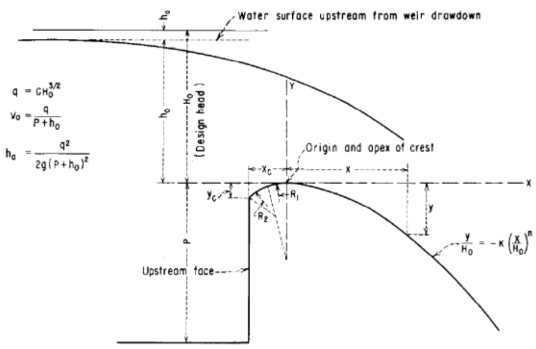


Figure 13. Definition of ogee parameters. (Small Dams Figure 9-20).

The Matlab code *draw_ogee.m* evaluates these parameters and draws the ogee profile together with the recommended blend radii between the crest and the approach slope upstream.

Design of Small Dams Fig 9-21 formulae for ogee profiles in terms of discharge coefficient, total head and invert height P :

$$v_a = \frac{Q}{L(P+h_o)} = \sqrt{2gh_a} \quad \text{and} \quad H = h_0 + h_a$$

Hence $2gh_a\left(P+H-h_a\right)^2 = \frac{Q}{L}$.

$$h\left(h^2 - 2Fh + F^2\right) - \frac{q}{2g} = 0 \quad \text{where} \quad h = h_a \quad \text{and} \quad F = P + H$$

This cubic can be solved In Matlab to find h_a .

More recently, Wahl (2008) has described a more accurate formula $y = -\frac{gx^2}{2v_0^2}$ where v_0 is the velocity at the horizontal section, e.g. for water over-topping a dam. Wahl notes that "when the crest is narrower than about 3H_{ovtop}, the situation tends toward the case of a sharp-crested weir, and the brink depth will begin to approach the critical depth" i.e. the velocity v_0 can be taken to be the critical velocity rather than a super-critical velocity further downstream.

Fairing 1 is designed to provide attached flow until levels rise above Q95 and fairing 3 becomes operational.

Fairing 2 is designed to provide attached flow until levels are high enough for the big CBS050 screen to deliver its rated capacity (stream flow roughly 27 litres/sec).

The design of fairing 3 is intended to ensure attached flow under all conceivable conditions.

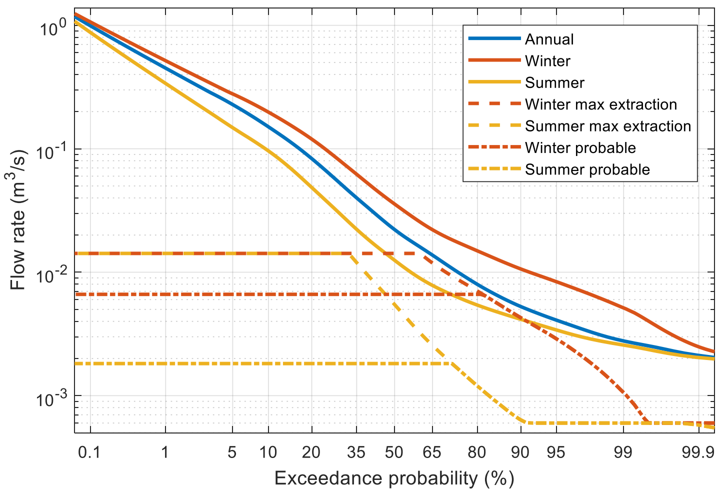


Figure 14. Monthly stream flow rate data from LowFlows grouped into Winter and Summer curves. Integrating to find the area under the line gives the time-mean flow rate (*lowflows_hyr8.m*). The "probable" curves (up to 6 kW in Winter, 1.5 kW Summer) may be more representative of actual usage. (*lowflows_hyr8.m*)

The LowFlows flow duration prediction (Figure 14) suggests extreme flow rates approaching 900 litres/second but this seems unlikely. The catchment area of 0.865 km^2 would give a stream flow rate which assymptoted to 100 litres/sec if there were 10 mm of rain every day. The nature of the terrain, with shallow moorland slopes and extensive bog (see the pre-app *HyR catchment photographs.pdf*), means that any rainfall runs off over an extended period – there is no flood surge passing downstream nor immediate run-off such as might happen with steep rocky slopes on Snowdon.

Examination of rainfall data on Snowdon (http://www.ecn.ac.uk/news/snowdon-december-2015-rainfall-record) shows that their record rainfall in December 2015 was equivalent to 45 mm per day.

Examination of the rainfall record at the Nant Peris gauging station and correcting for the annual rainfalls (3500mm versus 2410 at Hafod) http://www.ecn.ac.uk/data also suggests that extended periods in excess of 40 mm rain per day (a peak flow rate of 400 litres/sec) are unlikely.

Attempting to design a fairing for 400 litres/sec leads to a high fairing i.e. the CBS050 is pushed downwards. This involves a serious compromise – if the downstream water level is low enough to avoid screen flooding, the jump up to the plunge pool is uncomfortably high. Wahl (2008) notes that the flow over ogee spillways often stays attached with overtopping heads of 6-7 times the design head. A design head corresponding to 200 L/s stream flow has therefore been adopted instead of 400 L/s. This makes the fish passage up into the plunge basin much easier (see below). It should be noted that above 300 L/s the CBS050 screen is flooded and will should deliver some water regardless of whether the flow above it is separated or not.

The trajectories described by Small Dams and Wahl are compared in Figure 15 for a head corresponding to 200 litres/sec.

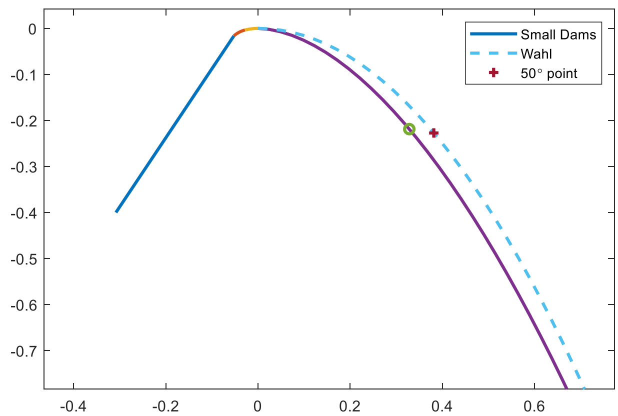


Figure 15. Comparison of SmallDams and Wahl formulae for ogee profile (H_3 = 0.245 m at 200 l/s, `draw_ogee('2:3', 0.4, 0.245)`).

Wahl's curve appears to provide an additional safety margin (slightly flatter curve) and has therefore been used in the current design. (It is also much easier to transfer a parabola into Fusion 360). The difference between the two curves is simply a stretch in the x-direction.

The crest heights resulting from the flow calculations have been matched to ogee curves giving the desired 50° slope on the screen plate (Figure 16).

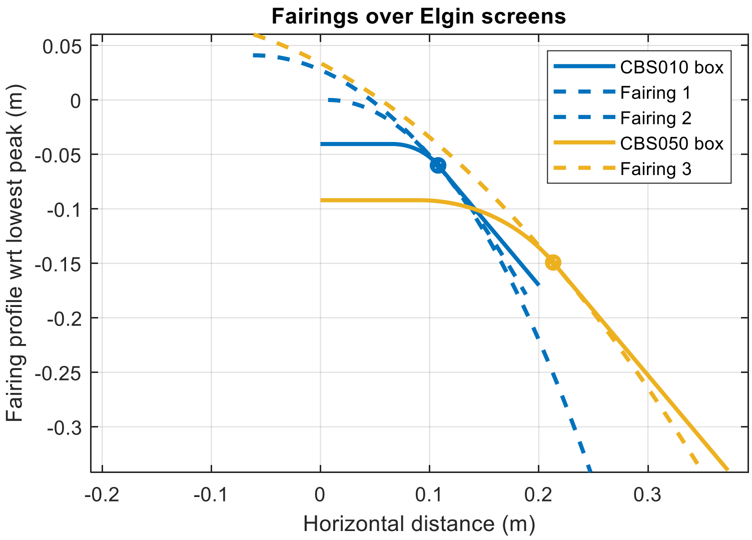


Figure 16. Collector box and ogee profiles (*run_weir_analysis.m*).

Design of the plunge pool basin

A cut-out (Fig. 17a) into the sloping rock slab shown in Figure 5 will form two sides of the plunge pool. A second cutout increases the plunge depth below the larger screen. The water depth in the basin is 320 mm at the Q95 condition.

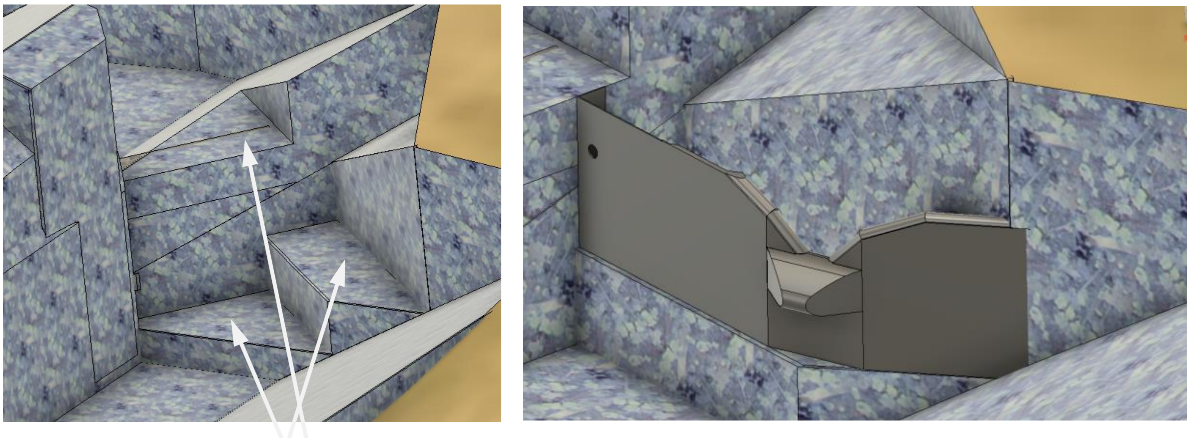


Figure 17 (a) Rock cut-outs, (b) Retaining plate forming a plunge pool. The ogee lip makes it easier for fish to jump up into this basin. The cut-out below the CBS050 screen removes the step seen on the left side of Fig. 5 to ensure an adequate plunge depth for fish passing down over the larger screen.

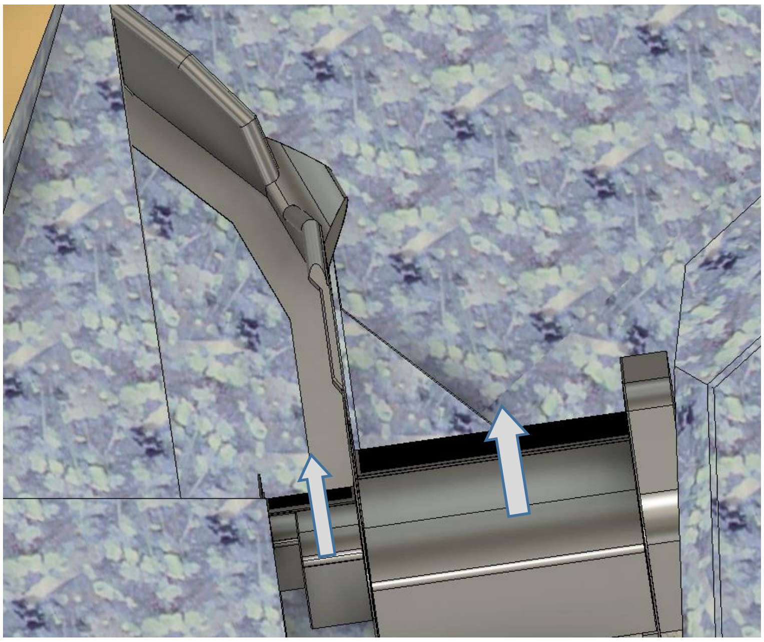


Figure 18. The plate lower edge folds inwards and is bolted and sealed to the rock. Fish passing downstream will always be below the metal edge, due to the screen cheek plates, and cannot accidentally land on top of it (ditto for fish passing over the large screen).

The basin lip has a shallow notch to provide a suitable depth of flow for jumping fish, even at low flow rates (Figure 19). The ogee lip avoids a sudden change in gradient to assist jumping fish at low flow rates (below Q70, 11 litres/s stream flow). (See section on fish jumps below). The upper edges of the retaining wall have a 40 mm tube welded around them to ensure a smooth profile.

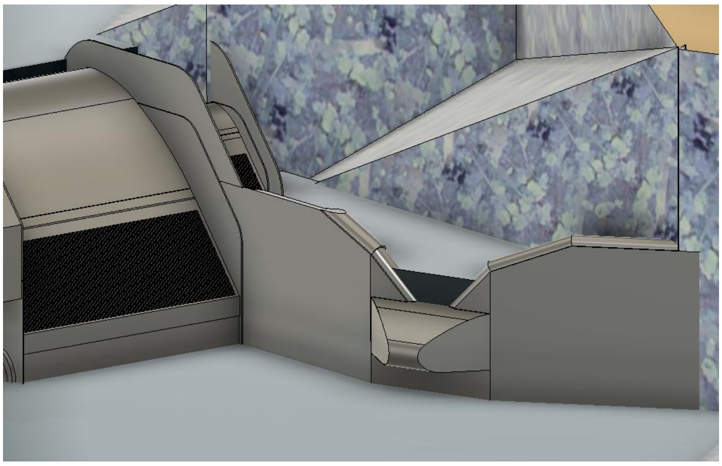


Figure 19. Basin notch profile (drawing HyR_181006C).

Downstream barrage design

Discussions with Dr Katrina Marshall suggested that indigenous brown trout could jump heights of up to 25 cm. To achieve this, the downstream water level is raised using a barrage weir (Figure 20); there are then three jumps up through the system, as shown in Figure 6. (The term “barrage” will be used to avoid confusion with the screens higher up which also constitute a slight weir).

There will be no large migratory fish spawning here because of the major waterfalls downstream (6-8 m vertically, see the pre-app geomorphology survey).

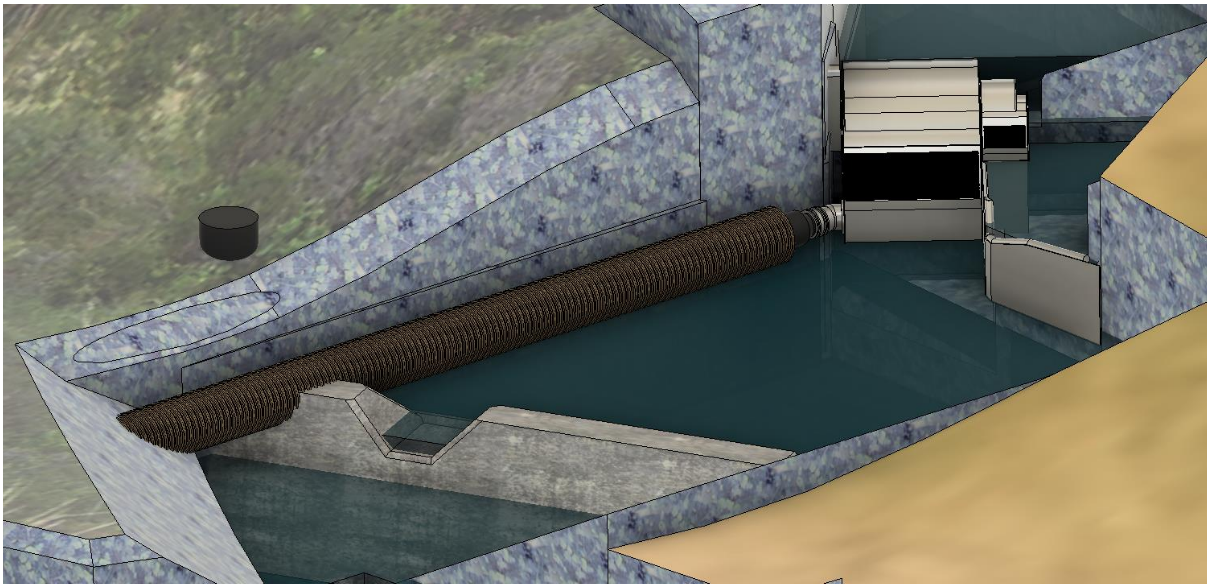


Figure 20. Barrage weir to raise water levels such that the jump up to the plunge basin is less than 25 cm. Drawing HyR_181006D.

The barrage weir has a notch to ensure an adequate depth of water for fish passage in the overflowing jet.

The notch also increases the sensitivity of flow measurements based on the water level above the barrage. The flow data tells the turbine control system whether more water is available; this is better than a “trial and error” system which would run the risk of sucking air into the penstock. The system response to a demand for more power is then either to open the spear valve further or to disconnect a load that cannot be driven with the current stream flows.

The notch is not a standard V-notch or Cipolletti weir profile so the flow versus head relationship will be obtained by calibration whilst commissioning. The stream level sensor will connect to the turbine control system via a fibre-optic link.

Barrage structural design

The barrage has been designed (in accordance with *Design of Small Dams*) to resist being dislodged by the forces on it. The worst-case load conditions have been assumed to be either:

- 900 litres/sec stream flow, equivalent to 90 mm/day rain continuing for enough days for the run-off to approach steady state (as opposed to one-off high rainfall incidents which experience suggests have a run-off spread over several days). This is the Q0.1 peak in the LowFlows prediction and gives a head of 383 mm above the top of the barrage.

- 236 litres/sec (Q5, 136 mm head above top of barrage) plus a magnitude 5.5 earthquake at a distance of 33 km. This magnitude is slightly more powerful than anything recorded in the last 400 years.

Figure 8.4 in Design of Small Dams suggests that a dynamic (as opposed to pseudo-static) analysis is unnecessary for earthquakes below magnitude 5.8 at 20 miles. Such a small, low aspect ratio construction will moreover have a high enough resonant frequency that vibrational deflections may be ignored.

North Wales has suffered a number of earthquakes of magnitude 5.2 – 5.4:

- Llyn peninsula 1984 (mag 5.4) 22.4 miles distance, depth 20.7 km
- Caernarfon 1852 (5.3) 20.6 miles
- Llanrwst 1780 (3.8) 10 miles
- Conwy, 2005 (3.3) 20 miles

The peak horizontal acceleration used in the design is $a_x = \lambda g$ with $\lambda = 0.07$ from the curve (SD Fig 8-5) for magnitude 5.5 at 33 km; the vertical acceleration is taken as 50% of the horizontal.

The earthquake water pressure formula for a pseudo-static analysis is $P_e = C\lambda wh$ where $w = \rho g$ and h = invert depth. At any depth y, the inertia of the water adds a horizontal force $V_e = 0.726 P_e y$ and a moment $M_e = 0.299 P_e y^2$

The coefficient C is shown in Small Dams Figure 8-5. Digitising the graph gives:

Face angle ψ from vertical	0°	15°	30°	45°	60°	75°
Coefficient C_m	0.73	0.63	0.52	0.41	0.30	0.165

$C = \frac{C_m}{2}\left(\frac{y}{h}\left(2-\frac{y}{h}\right)+\sqrt{\frac{y}{h}\left(2-\frac{y}{h}\right)}\right)$ such that $C = C_m$ at $y = h$.

Putting $f = \frac{y}{h}$, $g = 2f - f^2$ and $m = \frac{1}{2}\left(g+\sqrt{g}\right)$ gives $\int_0^1 m\,\mathrm{d}f = 0.726$ when integrated numerically, as per the Small Dams formula for V_e.

Similarly $\int_0^1 m(1-f)\,\mathrm{d}f = 0.2917$ (the 0.299 in the SmallDams M_e formula appears to be a slight misprint). If the face is inclined at angle ψ to the vertical (with appropriate choice C_m value) the vertical force component will be $V_{e,v} = V_{e,h}\tan\psi$ and the moment about the bottom end of the face will be $M_{e,\psi} = \frac{M_{e,0}}{\cos^2\psi}$

The concrete will be keyed into a trough 80 mm deep in the bedrock to avoid any risk of sliding. The risk of roll-over due to water pressure has been assessed by taking moments about the downstream toe point (T) of the dam using the parameters shown in Figure 24:

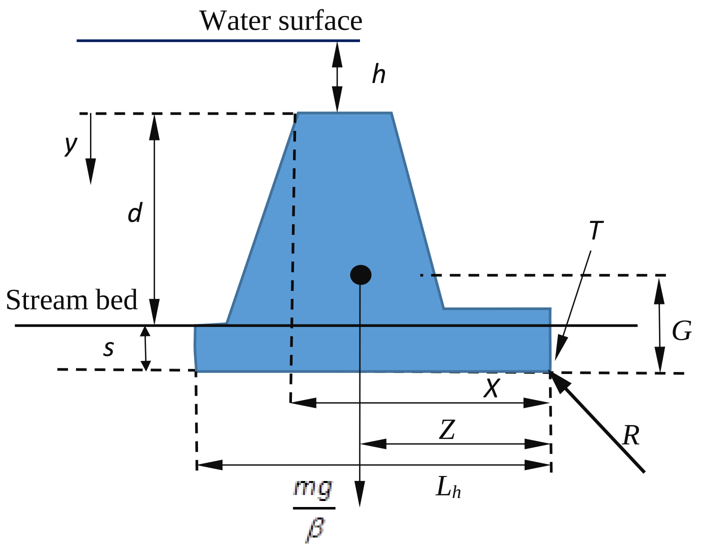


Figure 21. Nomenclature for moment calculations. Each dimension has an upstream and downstream value relative to a common toe point *T*.

A safety factor β relative to the mass m per metre width of dam is defined such that a mass $\frac{m}{\beta}$ would be just sufficient to prevent toppling (zero net moment) under the maximum loading condition and with the dam supported purely by a reaction force *R* passing through point *T*. A safety factor <1 would imply that the dam was too light and reliant on the adhesion of the concrete to bedrock and/or the use of rock anchors to hold it in place.

Water pressure $P(y) = \rho g(y+h)$

Upstream face at angle ψ clockwise from vertical; define $k = \tan\psi$

The clockwise moment about *T* (per unit width) due to the water pressure on either dam face is

$$M_{p,f} = f\int_0^d \rho g(y+h)(d+s-y)\mathrm{d}y - f\int_0^d \rho g(y+h)(X+ky)k\mathrm{d}y$$
$$= f\rho g\int_0^d (y+h)\left(d+s-kX-\left(1+k^2\right)y\right)\mathrm{d}y$$
$$= f\rho g\int_0^d (y+h)(A-By)\mathrm{d}y = f\rho g\int_0^d Ah+(A-Bh)y-By^2\mathrm{d}y$$
$$= f\rho g\left(Ad\left(h+\frac{d}{2}\right)-Bd^2\left(\frac{h}{2}+\frac{d}{3}\right)\right)$$

where $A = d+s-kX$, $B = 1+k^2$, $Y = d+s$ and f = 1, -1 for upstream and downstream faces.

If the foundation is sunk below stream bed level ($s \neq 0$) the hydrostatic pressure is assumed for simplicity to increase down to the foundation bed plane; this may be over-pessimistic. The moment over these vertical sections is

$$M_{p,s} = f\int_d^Y \rho g(y+h)(d+s-y) = f\rho g\int_d^Y h(d+s)+(d+s-h)y-y^2\mathrm{d}y$$
$$= f\rho g\left(hs(d+s)+\frac{1}{2}(d+s-h)\left(Y^2-d^2\right)-\frac{1}{3}\left(Y^3-d^3\right)\right)$$

Under the dam it is assumed (in accordance with *Small Dams*) that the pore pressure of water beneath the concrete varies linearly between upstream P_1 and downstream P_2. The clockwise moment about *T* due to this pressure over the distance $L=\sqrt{L_h^2+\left(s_1-s_2\right)^2}$ will be $M_{pore}=L^2\left(\frac{P_1}{3}+\frac{P_2}{6}\right)$

Force calculations based on hydrostatic pressure give

Downstream thrust $H=\left(P_1-P_2\right)\left(s_2-s_1\right)+\sum f\rho g\int_0^Y (h+y)dy=\left(P_1-P_2\right)\left(s_2-s_1\right)+\sum f\rho g\left(hY+\frac{Y^2}{2}\right)$

Vertical down-thrust $V=-\left(\frac{P_1+P_2}{2}\right)L_h+\sum \rho g\int_0^d (h+y)kdy=-\left(\frac{P_1+P_2}{2}\right)L_h+\sum \rho gk\left(hd+\frac{d^2}{2}\right)$

Condition:	900 litres/sec	236 litres/sec + earthquake	32 L/s + earthquake
Safety factor β :	2.09	2.08	2.08
Min. coeff of friction (if unkeyed) – ref. only	0.16	0.32	0.32

(*barrage_stability.m*)

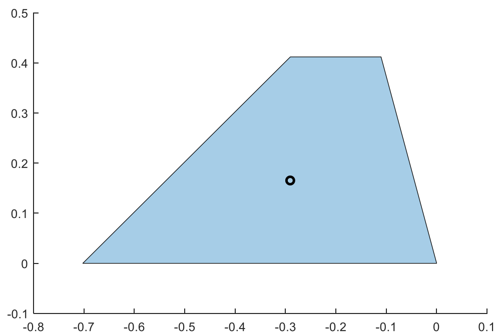


Figure 22. Dam cross-section at deepest point (stream flow from left to right).

Predictions of water levels and fish jump heights

The height of the Coanda screen fairings relative to the plunge pool basin and barrage involves some compromise between conflicting design aims:

- avoid excessive increases in upstream water level
- avoid excessive excavation into bedrock
- ensure brown trout can pass up and down-stream
- ensure proper operation of the Coanda screens and facilitate inspection and cleaning by keeping the screens unflooded i.e. above the downstream water level.

After many iterations, a combination of crest profiles and heights has been obtained that manages to satisfy these requirements. The stream levels are shown in Figure 23 as a function of upstream flow rate. A maximum extraction of 2 L/s has been assumed for this graph: this is probably more typical, on average, than the maximum possible extraction.

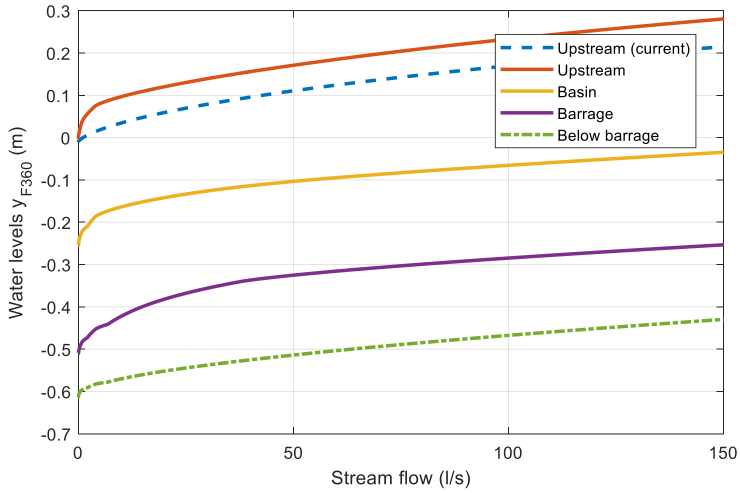


Figure 23. Water levels (based on head *H* in a zero velocity pool) as a function of stream flow. The datum here is the origin in Fusion 360.

YouTube video footage (Figure 24) shows brown trout leaping into the flow over a weir and attempting to swim up over it; the weir height in this case appears to be about 1.5 m (and about 3× fish length) so they cannot simply jump it.

Figure 24. Brown trout swimming up a weir (https://www.youtube.com/watch?v=tMvexb97TqA). The flow in this video separates from the edge of an inclined flat plate. The fish find it difficult to get past the lip, especially at this low flow condition which leads to a rapid change in gradient at the lip. An ogee profile with attached flow should be easier to traverse.

The water accelerates as it falls: as a design rule, the fish should be able to jump up into water which is moving sufficiently slowly that they can swim away upstream. Small brown trout can reach burst speeds of 9.87 lengths/second (fishbase.org) for a 24 cm fish.   Fish in Afon y Foel are likely to reach 17-18 cm in length (conversation with Joel Rees-Jones); on this basis, a 17 cm fish can swim at 1.7 m/s. This is a conservative estimate since speed is proportional to tail beating frequency: the maximum frequency attainable decreases with increasing size of the specimen (Bainbridge, 1958) and might be slightly higher for a 17 cm fish than for the 24 cm fish recorded in the FishBase data.

Jump heights have therefore been calculated from the downstream water surface up to the point with speed 1 m/s (safely below the 1.7 m/s swimming speed). This is approximately 5 cm lower than the surface of a zero-velocity pool far upstream.

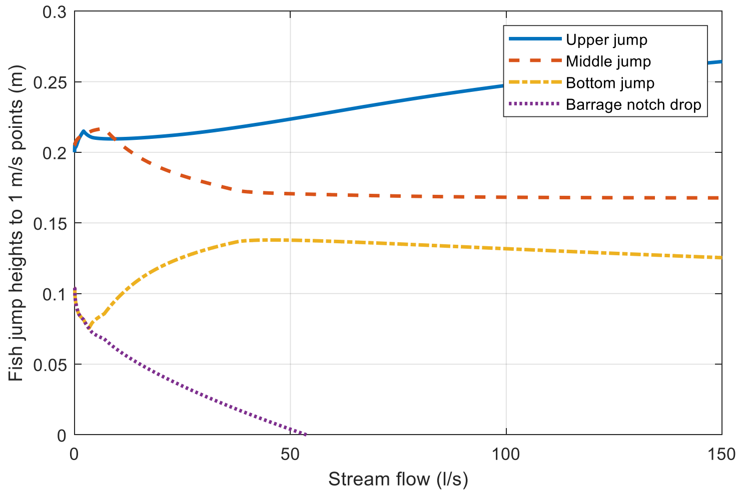


Figure 25. Fish jump heights. The notch profiles have been chosen to provide similar increases in water level (with increasing flow rate) above each jump, thereby maintaining a roughly constant jump from one pool to the next irrespective of flow rate. The upper jump is over Fairing 2; the jump over Fairing 1 would be 0.041 m lower.

The base of the barrage notch is approximately 8 cm above the downstream water level at low flow rates (4 L/s, Fig. 25); at higher flow rates the downstream level rises until the notch base becomes submerged at about 53 L/s. The notch profile has been chosen both to maintain a suitable jump height up into the plunge basin and to facilitate stream flow rate estimation based on the water level in front of the barrage. The maximum possible screen extraction should be achievable for stream flows above 26 L/s so accurate measurement of flows beyond 50 L/s is unnecessary.

As water levels increase, the screens start to be submerged in the downstream pools, Figure 26.

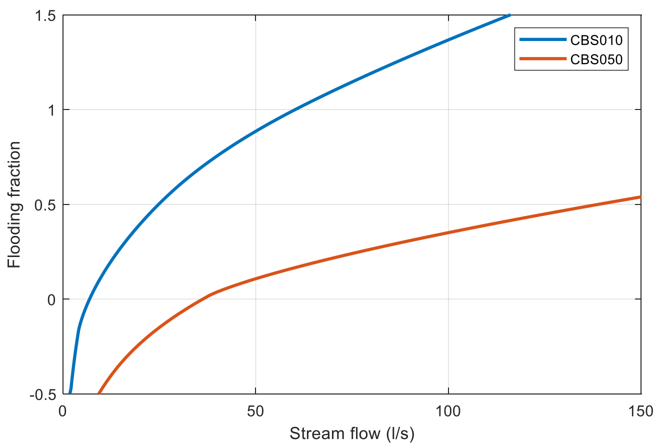


Figure 26. The CBS010 screen starts to flood at 7 L/s and is completely flooded above 60 L/s. The CBS050 starts to flood at 36 L/s and is completely flooded at 300 L/s. (These figures are approximate

because the weir flow calculation assumes critical flow over the crest, with no allowance for back-pressure due to high water levels downstream).

A typical profile for water flowing over an ogee fairing is shown in Figure 27.

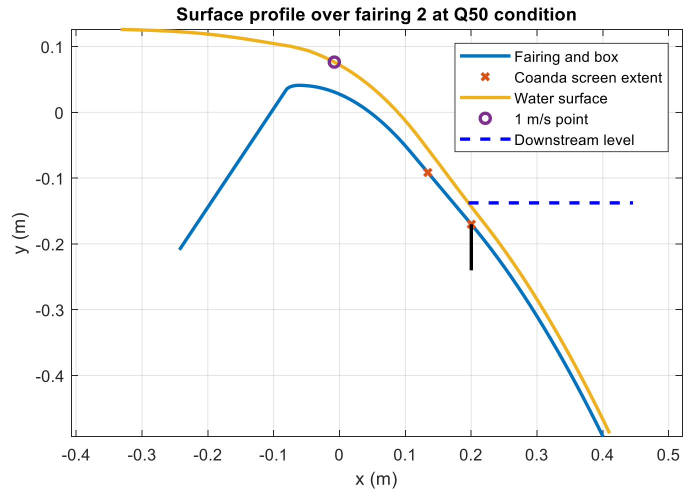


Figure 27. Jump height definitions, showing flow over fairing 2 with an upstream level equivalent to Q50 flow (22 litres/sec) and 0.6 L/s extraction over the length of the screen.

The water depth d normal to the fairing in Figure 27 has been calculated using a continuity equation:

$$C_d L\sqrt{g}\left(\frac{2}{3}H\right)^{\frac{3}{2}} - Q_{ext} = Ld\sqrt{2g\left(A - d\cos\theta\right)} \text{ where } A - d\cos\theta = H + \left(y_0 - y\right) - d\cos\theta \text{ (Figure 28)}$$

Re-arranging: $$\left(C_d\sqrt{g}\left(\frac{2}{3}H\right)^{\frac{3}{2}} - \frac{Q_{ext}}{L}\right)^2 = 2g\left(A - d\cos\theta\right)d^2$$

$$2g\cos\theta d^3 - 2gAd^2 + \left(C_d\sqrt{g}\left(\frac{2}{3}H\right)^{\frac{3}{2}} - \frac{Q_{ext}}{L}\right)^2 = 0$$

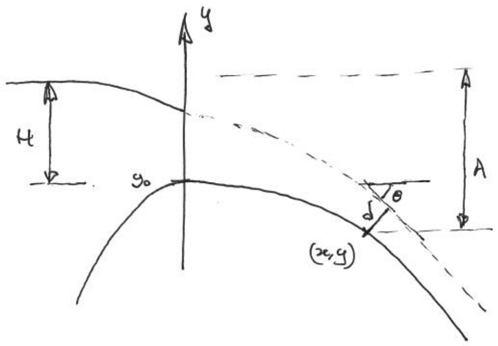


Figure 28. Parameter definitions for nappe profile calculation.

Below the screen, the water separates into a free jet. The lower surface follows the curve $y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0}$ (Wahl, 2008) where v is based on the upper surface level (from the above calculation.

The water profile in front of the critical velocity point in Figure 27 has been estimated using the empirical correlation (Goodarzi, 2012):

$Y = a - b\tanh\left(\frac{X-c}{d}\right)$ where $Y = \frac{y}{H}$ and $X = \frac{x}{H}$. Coefficients a,b,c and d are tabulated in their paper; the horizontal position of the curve has been matched to the water surface calculated above.

The fairing profile in front of the crest in Figure 27 has been designed using the parameters given in Design of Small Dams, Figure 9-21. The velocity head h_a (an input parameter to their correlations) has been derived as follows:

$v_a = \frac{Q}{L(P+h_o)} = \sqrt{2gh_a}$ and $v_a = \frac{Q}{L(P+h_o)} = \sqrt{2gh_a}$

Hence $2gh_a\left(P+H-h_a\right)^2 = \left(\frac{Q}{L}\right)^2$, giving a cubic $h_a\left(h_a^2 - 2Fh_a + F^2\right) - \frac{q^2}{2g} = 0$ where $q = \frac{Q}{L}$ and $F = P + H$

Datum heights

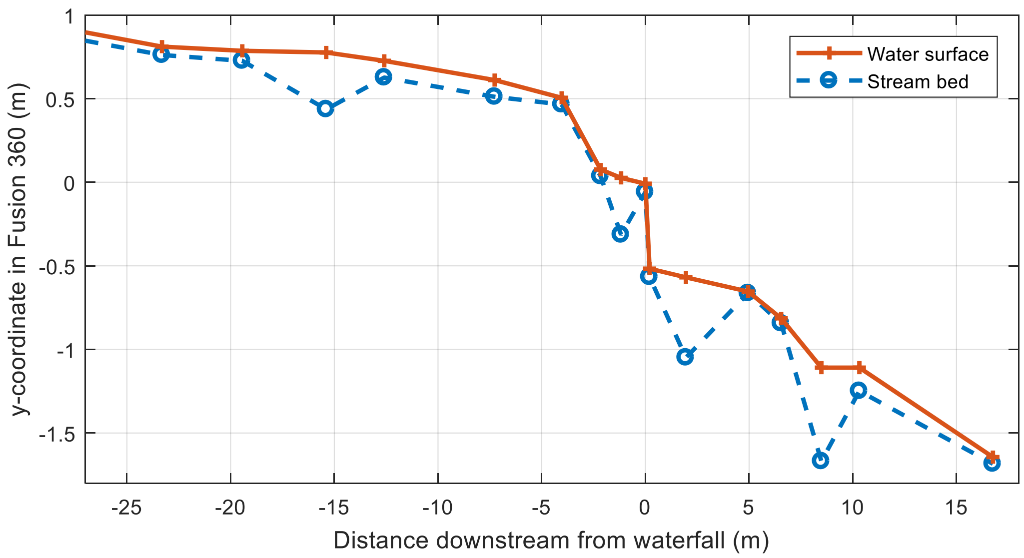
Two bank heights were measured as datum points in the survey. The contour lines in drawing HyR_181006A.pdf suggest that these are at a mean height above sea level of 423.25 m. Heights relative to this datum have been determined from the Fusion 360 model (Table 1).

	yF360	mASL
Bank markers (mean)	1.552	423.25
Top of rock on corner above screens	0.853	422.551
Waterfall lip (lowest corner)	-0.059	421.639
Fairing 1 crest	-0.003	421.695
Fairing 2 crest	0.038	421.736
Fairing 3 crest	0.075	421.773
Plunge basin (bottom of notch)	-0.254	421.444
Barrage (bottom of notch)	-0.510	421.188
Stream bed above screens	-0.314	421.384
Plunge basin bed	-0.568	421.130
Stream bed above barrage/below CBS050	-1.049	420.649

Table 1. Datum heights (metres) for each weir stage.

Further details of the crest profiles are provided in the Matlab code (*weir_defs1.m*). The relative heights of fairings 1, 2 and 3 are 0, 0.041, 0.078 m respectively.

The stream bed heights and water depths measured in the survey are shown in Figure 29.



The deep sections are pools. These tend to slope on one side due to the inclination of the rock strata; in the flow direction the bottom Is largely level. The rapids 3-4m upstream of the waterfall, with a drop of 0.5 m, mean that any slight change in water level due to the screen assembly will not have any impact further upstream.

Eel passage

Migratory eels may be unable to pass from the sea up to this level. The depleted reach section has numerous large waterfalls and passes through a steep sided gorge, which can be challenging on foot: the only way of bypassing these waterfalls would be to make a detour of some 400 m and pass above the gorges. (See *HyR geomorphology survey 4.pdf* and *Obstacles to fish.pdf*).

An eel canal will however be provided to make it easier for any eels reaching the vicinity to come out of the stream just below the barrage and pass overland to the section above, Figure 30.

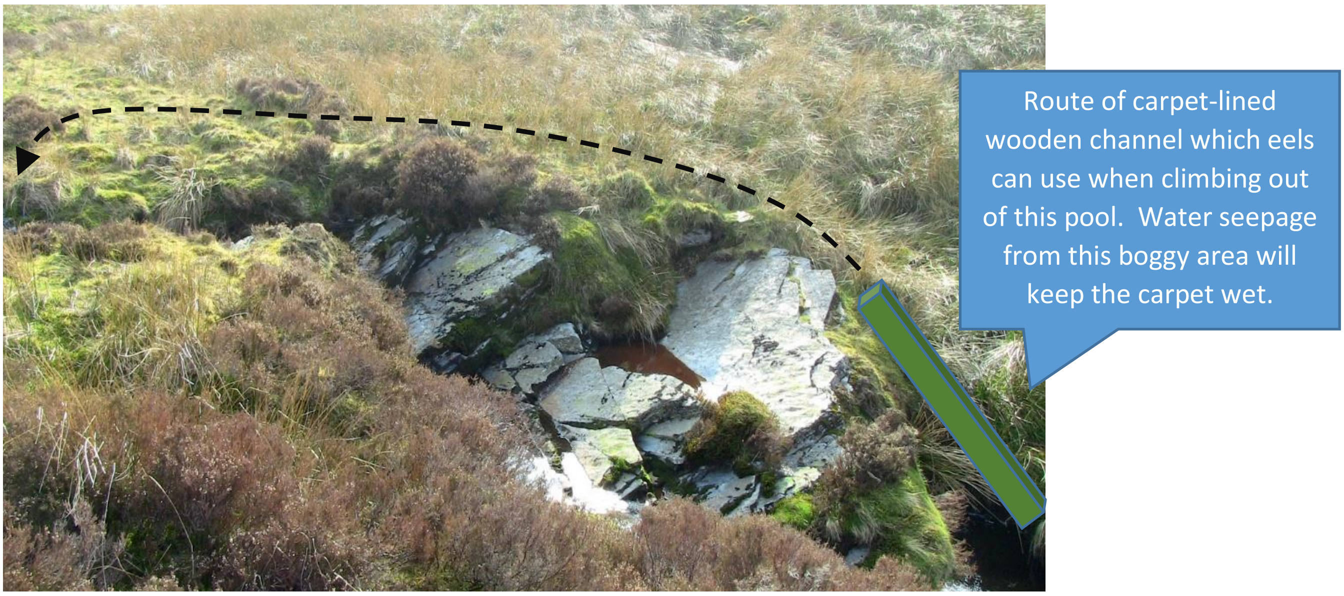


Figure 30. Possible eel passage route. See also drawing HyR_18Nov01A.pdf and Figure 6. The channel is 200 mm wide.

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October 2018.